УДК 629.429.3:621.313 **doi: 10.20998/2313-8890.2025.02.02**

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GENERAL APPROACHES TO CREATING A MATHEMATICAL MODEL OF A SYNCHRONOUS REACTIVE MOTOR WITH PERMANENT MAGNETS FOR ELECTRIC TRANSPORT

Abstract. *The article considers promising methods for creating mathematical models of traction synchronous-reactive motors with permanent magnets, which makes it possible to take into account the geometric parameters of the location of magnets in the motor excitation system. According to the results of numerical experiments on the calculations of the magnetic field by the finite element method in the plane-parallel formulation of the problem and subsequent regression analysis, five-dimensional polynomial dependences of the derivatives of flux linkages and torque on current and angular coordinate were obtained, which make it possible to identify a generalized mathematical model of a synchronous reactive traction motor, both with a non-sectioned and sectioned rotor, for a metro car. When conducting regression analysis, a rational order of the function approximating the flux linkage of the motor phases and its derivatives, as well as the electromagnetic torque, was determined. For a synchronous jet motor with permanent magnets, the following are determined: the number of harmonics is 7, the degree of the polynomial is 3 with a maximum deviation of 2.62 % for a non-sectioned rotor and 2.79 for a sectioned one.*
Keywords: *traction motor, permanent magnets, electric transport, excitation system, polynomial dependences of flux linkages.*

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ЗАГАЛЬНІ ПІДХОДИ ДО СТВОРЕННЯ МАТЕМАТИЧНОЇ МОДЕЛІ СИНХРОННО-РЕАКТИВНОГО ДВИГУНА З ПОСТІЙНИМИ МАГНІТАМИ ДЛЯ ЕЛЕКТРОТРАНСПОРТУ

Анотація. *Стаття розглядає перспективні методи щодо створення математичних моделей тягових синхронно-реактивних двигунів з постійними магнітами, яка надає можливість врахування геометричних параметрів розташування магнітів у системі збудження двигуна. За результатами чисельних експериментів щодо розрахунків магнітного поля методом скінчених елементів у плоскопаралельній постановці задачі і подальшого регресійного аналізу отримано п'ятивимірні поліноміальні залежності похідних потокозчеплень та моменту за струмом та кутовою координатою, які дають можливість ідентифікувати узагальнену математичну модель синхронного реактивного тягового двигуна, як з несекціанованим так і секціанованим* *ротором, для метровагону. При проведені регресійного аналізу визначено раціональний порядок функції, що апроксимує потокозчеплення фаз двигуна та його похідних, а також електромагнітного моменту. Для синхронного реактивного двигуна з постійними магнітами визначено: число гармонік 7, ступінь полінома – 3 при максимальному відхиленні 2,62 % для несекціанованого ротору та 2,79 – для секціанованого.*

Ключові слова: *тяговий двигун, постійний магніти, електротранспорт, система збудження, поліноміальні залежності потокозчеплень.*

Introduction. Today, world practice involves the use of traction electric drives with asynchronous motors in railway rolling stock, as well as in industrial and urban transport. In Ukraine, powerful asynchronous electric drives are used in urban electric transport and on main railways [1].

The main advantages of a traction asynchronous electric drive are high energy efficiency, optimal mass-dimensional characteristics, reliability, simple design and long service life. However, the need to reduce energy consumption and increase the resource of rolling stock poses the scientific and technical community the task of further improving asynchronous traction electric drives, as well as research and development of alternative types of electric drives [2].

One of such alternative approaches is the use of synchronous motors with excitation from permanent magnets [3]. However, a significant mass of high-coercive magnets significantly increases the cost of production of such electric motors.

Analysis of previous research. One of the directions of creation of promising energy-saving technologies for metro cars is the use of traction drives based on synchronous traction motors. This type of traction motors provides high efficiency indicators at partial power of the traction drive and high accelerations during acceleration and braking of the train [4].

Let us consider the main provisions on the creation of the main element of the mathematical model of the traction drive – a synchronous motor. Due to the fact that high energy indicators of synchronous motors are achieved due to the use of a magnetic system with a complex distribution of magnetic flux, the mathematical model must take into account the geometric features of the rotor and stator of the motor [5]. Such a model is created on the basis of the level of a generalized mathematical model. The motor can be represented as a system having N_{in} electrical and Min mechanical coordinates (inputs). For most synchronous motors of modern and promising types, the number of mechanical inputs (shafts, armatures) is equal to one. Therefore, in the following $M_{in} = 1$ (Fig. 1).

Statement of the task. The purpose of the work is to develop and identify the parameters of a mathematical model of synchronous jet traction motors with permanent magnets for a metro car.

Tasks of the work. 1. Based on the Lagrange equation for an electromechanical system, develop a mathematical model of a three-phase traction motor. 2. Identify a mathematical model for the case of a synchronous jet traction motor with sectioned and non-sectioned rotors.

Research results. Let us choose the following generalized energy parameters of the engine: generalized coordinate ς_k, generalized velocity ς'_k, generalized force impulse p_k, generalized force f_k. For a mechanical system, the generalized coordinates coincide with the mechanical description, the generalized coordinate is the angle of rotation of the rotor, the generalized velocity is the angular velocity, and the force is the moment. When choosing electrical coordinates, we will apply the method of electromechanical analogy [4, 5] for electromechanical systems (Fig. 1).

The electrical energy of an electromechanical system, which is stored in the system, consists of potential energy and kinetic coenergy. This means that if we define the charge q as a generalized coordinate ς_k, then the electrical stored energy will be defined as potential.

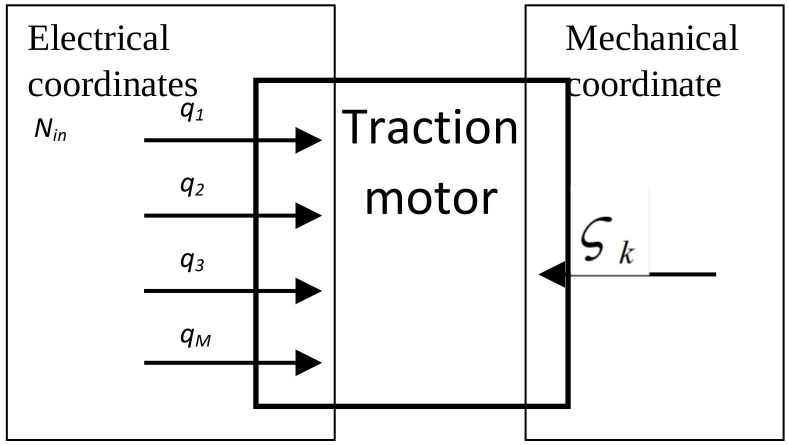


Fig. 1 – Structural diagram of the mathematical model of a traction motor

Considering the action of a non-conservative force Q_k on the k-th coordinate ς_k together with the conservative forces of the system, according to d'Alembert's principle, for dynamic equilibrium the total sum of all forces, including non-conservative ones, must be zero. The Lagrange equation of the second kind for the system has the following form

$$\frac{\partial L}{\partial \varsigma_k} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varsigma}_k}\right) - \left(\frac{\partial F}{\partial \dot{\varsigma}_k}\right) + Q_k = 0, \quad (1)$$

where L – Lagrangian force function, F – Rayleigh function describing the losses in the system.

After determining the generalized coordinates, we choose the Lagrangian force function, or Lagrangian, which we will use to obtain the equations of motion. The Lagrangian is defined as the difference between the kinetic co-energy and the potential energy V, i.e.

$$L = T - V. \quad (2)$$

In traction motors there is only one mechanical coordinate, so the main energy parameters can be presented in Tab. 1.

Table 1 – Main energy parameters of a synchronous traction motor

Energy parameter	Electrical parameters	Mechanical parameters
k	$1,2,...M$	$M+1$
ς_k	$q_1, q_2, ..q_M$	γ
ς'_k	$i_1, i_2, ..i_M$	ω
p_k	$\psi_1, \psi_2, ..\psi_M$	$J\omega$
$-f_k$	0	0
Q_k	U_1, U_2,.. U_M	M_r

Where k – generalized coordinate number, M– number of electrical 1,2..M coordinates (windings) of a synchronous traction motor, $q_1, q_2, ..q_M$ – charges in the windings of a synchronous traction motor, $i_1, i_2, ..i_M$ – generalized velocities – currents in the windings 1,2..M, 1,2..M – generalized pulses – flux linkage in the windings of a synchronous traction motor, U_1, U_2,.. U_M – generalized non-conservative forces – voltages applied to the windings of a synchronous traction motor, γ – generalized mechanical coordinates – displacement of the moving part or the angle of rotation of the rotor of a synchronous traction motor, ω – generalized mechanical velocities – angular velocity of rotation of the rotor of a synchronous traction motor, J– moment of inertia of the rotor of a synchronous traction motor, M_c– generalized non-conservative forces – force and moment of resistance.

Through generalized variables, we write the expressions for kinetic co-energy and potential energy for the conservative part of the system according to the following equations:

- kinetic co-energy

$$T = \int_{0,\dots,0}^{q_1,\dots,q_{M+1}} \sum_{k=1}^{M+1} p_k(\varsigma_1,\dots,\varsigma_{M+1};\dot{\varsigma}_1,\dots,\dot{\varsigma}_{M+1};t)d\dot{\varsigma}_k \qquad (3)$$

- potential energy

$$V = \int_{0,\dots,0}^{q_1,\dots,q_{M+1}} \sum_{k=1}^{M+1} - f_k(\varsigma_1,\dots,\varsigma_{M+1};t)d\varsigma_k \tag{4}$$

Taking into account the generalized coordinates and expressions for kinetic coenergy and potential energy for the conservative part of the system according to equations (1) and (2) establishes

$$T = \frac{1}{2}\cdot J\omega^2 + \int_0^{i_1} \Psi_A(i_1', i_2, \dots, i_M, \gamma)di_1' + \\ + \int_0^{i_2} \Psi_B(0, i_2', \dots, i_M, \gamma)di_2' + \dots + \int_0^{i_M} \Psi_D(0,0,\dots,i_M', \gamma)di_M'. \tag{5}$$

$$V = 0 \tag{6}$$

The kinetic coenergy for the magnetic field, according to the d'Alembert principle, is carried out by integrating over different contours, so the sequence of integration of the phase flux linkages can be any.

The conservative Lagrangian is equal to the kinetic coenergy

$$L = T\,. \tag{7}$$

The relay loss function is the sum of the ohmic losses in the motor windings and the friction losses in the bearings, which depend on the speed of rotation in the motor and the drive.

$$F = \frac{1}{2}\left(\sum_{k=1}^{M} r_k i_k^{\,2} + \alpha\omega^2\right), \tag{8}$$

where α –friction coefficient in the bearings of the traction motor and drive.

Transforming expressions (1), (5)-(8), taking into account the corresponding derivatives in generalized coordinates and velocities, we obtain the equation of the motor windings for the electrical coordinates

$$U_k - \frac{d\Psi_k(i_1, i_2, \dots, i_M, \gamma)}{dt} - r_k i_k = 0, \tag{9}$$

and for the mechanical coordinate – the moment equation

$$\frac{\partial\left[\int_0^{i_1}\Psi_1(i_1',i_2 i_3,\gamma)di_1' + \int_0^{i_2}\Psi_2(0,i_2' i_3,\gamma)di_2' + \int_0^{i_M}\Psi_M(0,0,i_3',\gamma)di_3'\right]}{\partial\gamma} - \quad (10)$$

$$-J\frac{d\omega}{dt} - \alpha\omega - M_r = 0$$

Considering the most common three-phase circuit of semiconductor converters, we consider the following model of a synchronous traction motor for the case of the number of stator phases M=3. We consider that the electromagnetic torque of the motor can be determined by the expression [4, 5]

$$M_m = \frac{\partial\left[\int_0^{i_1}\Psi_1(i_1',i_2 i_3,\gamma)di_1' + \int_0^{i_2}\Psi_2(0,i_2' i_3,\gamma)di_2' + \int_0^{i_M}\Psi_M(0,0,i_3',\gamma)di_3'\right]}{\partial\gamma}. \quad (11)$$

Since in our case the flux linkage is a complex function that depends on all generalized coordinates, the general derivatives of the flux linkages can be transformed into the following form:

$$\frac{d\Psi_k}{dt} = \sum_{n=1}^{M}\left(\frac{\partial\Psi_1}{\partial i_n}\cdot\frac{di_n}{dt}\right) + \frac{\partial\Psi_1}{\partial\gamma}\cdot\frac{d\gamma}{dt} \quad (11)$$

Let us substitute (11) into the system of equations (9) and perform the level of the motor's electrical circuits in the following form:

$$\sum_{n=1}^{M}\left(\frac{\partial\Psi_1}{\partial i_n}\cdot\frac{di_n}{dt}\right) + \frac{\partial\Psi_1}{\partial\gamma}\cdot\omega + r_1 i_1 = U_1; \sum_{n=1}^{M}\left(\frac{\partial\Psi_2}{\partial i_n}\cdot\frac{di_n}{dt}\right) + \frac{\partial\Psi_2}{\partial\gamma}\cdot\omega + r_2 i_2 = U_2;$$
$$\sum_{n=1}^{M}\left(\frac{\partial\Psi_3}{\partial i_n}\cdot\frac{di_n}{dt}\right) + \frac{\partial\Psi_3}{\partial\gamma}\cdot\omega + r_3 i_3 = U_3. \quad (12)$$

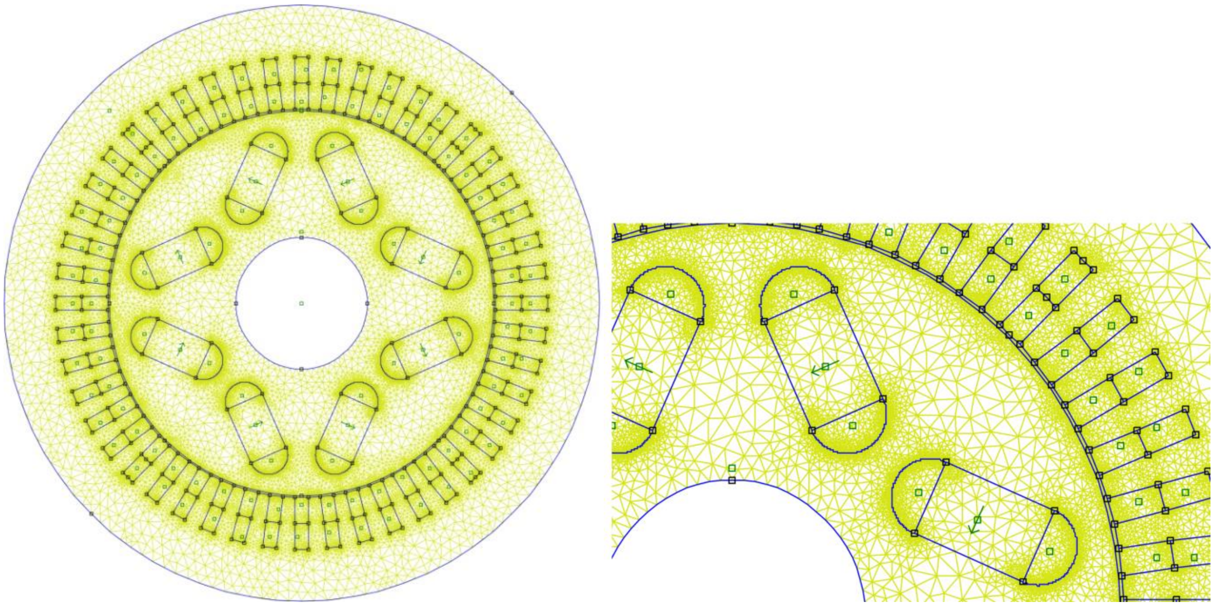
To transform equations (12) of the motor's electrical circuits into the form of a Cauchy problem, we solve the system of equations with respect to the $\frac{di_1}{dt},\frac{di_2}{dt},\frac{di_3}{dt}$ derivatives, solving which by Cramer's method we obtain an expression for the derivative of the kth current in the form

$$\frac{di_k}{dt}=\frac{\begin{vmatrix} \frac{\partial \Psi_1}{\partial i_1} & \frac{\partial \Psi_1}{\partial i_2} & \left[U_1-\frac{\partial \Psi_1}{\partial \gamma}\cdot\omega-r_1 i_1\right]_{(j=k)} & \frac{\partial \Psi_1}{\partial i_M} \\ \frac{\partial \Psi_2}{\partial i_1} & \frac{\partial \Psi_2}{\partial i_2} & \left[U_2-\frac{\partial \Psi_2}{\partial \gamma}\cdot\omega-r_2 i_2\right]_{(j=k)} & \frac{\partial \Psi_2}{\partial i_M} \\ \frac{\partial \Psi_3}{\partial i_1} & \frac{\partial \Psi_3}{\partial i_2} & \left[U_M-\frac{\partial \Psi_3}{\partial \gamma}\cdot\omega-r_3 i_3\right]_{(j=k)} & \frac{\partial \Psi_3}{\partial i_3} \end{vmatrix}}{\begin{vmatrix} \frac{\partial \Psi_1}{\partial i_1} & \frac{\partial \Psi_1}{\partial i_2} & \frac{\partial \Psi_1}{\partial i_3} \\ \frac{\partial \Psi_2}{\partial i_1} & \frac{\partial \Psi_2}{\partial i_2} & \frac{\partial \Psi_2}{\partial i_3} \\ \frac{\partial \Psi_M}{\partial i_1} & \frac{\partial \Psi_M}{\partial i_2} & \frac{\partial \Psi_3}{\partial i_3} \end{vmatrix}}, \tag{13}$$

where (j=i) – column j number corresponding to the current k number .

Equation (13) is a differential equation for the -th current, which is presented in the form of a Cauchy problem. However, the use of equations in this form for a mathematical model requires significant computational resources, because at each step of solving the system of equations of the form (13), it is necessary to find the determinants of the components of the numerator and denominator, the elements of which depend on the state of the magnetic system of the motor. Therefore, it is proposed to reduce the order of the mathematical model without simplifying the determinants.

The model parameters have been identified for the most common type of synchronous traction motor – a synchronous reactive motor with permanent magnets. Fig. 2 shows the calculation area for a synchronous reactive motor with a non-sectioned rotor, and Fig. 3 – with a sectioned one.



a b

Fig. 2 Finite element mesh of a synchronous jet motor with non-sectioned permanent magnets: a – general view; b – permanent magnet zone

Each of these types has its advantages and disadvantages, however, the decision on their further application is possible only on the basis of a comparative analysis of dynamic characteristics. The parameters of all engines are the same and are given in (tab. 1).

Identification of model parameters (13) includes establishing the dependence between the flux linkages of the phases Ψ_A, Ψ_B, Ψ_C, and on the one hand, generalized coordinates, and their derivatives on the other. The elements of the magnetic systems of the considered synchronous engines due to high use have areas with saturation significantly greater than *2 Tesla*. Therefore, to obtain the dependences of the flux linkages on the angle of rotation of the rotor, a set of digital experiments is carried out to calculate the magnetic field. To obtain the value of the flux linkages, it is proposed to carry out calculations of the magnetic field in a two-dimensional formulation using the finite element method in the plane-parallel problem formulation. According to the results of the magnetic field analysis, the value of the flux linkages of the engine phases and the value of the electromagnetic moment are determined.

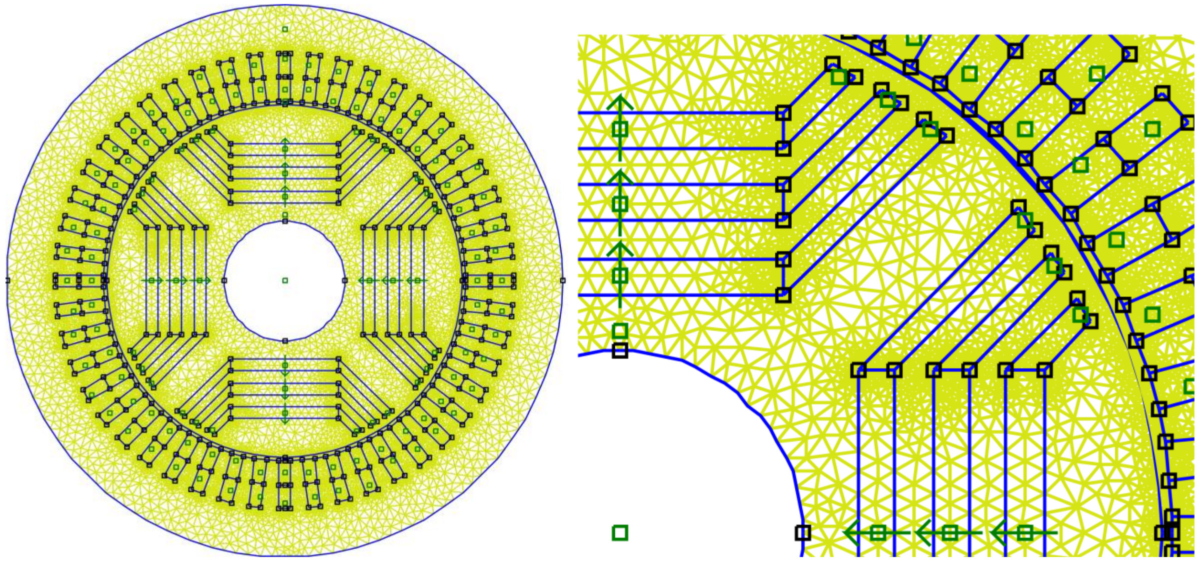


Fig. 3 Finite element mesh of a synchronous jet engine with sectioned permanent magnets: a – general view; b – permanent magnet zone

The angle of rotation of the rotor should be changed rationally in the interval from 0 to $2\pi / p$,, where *p* – number of pole pairs.

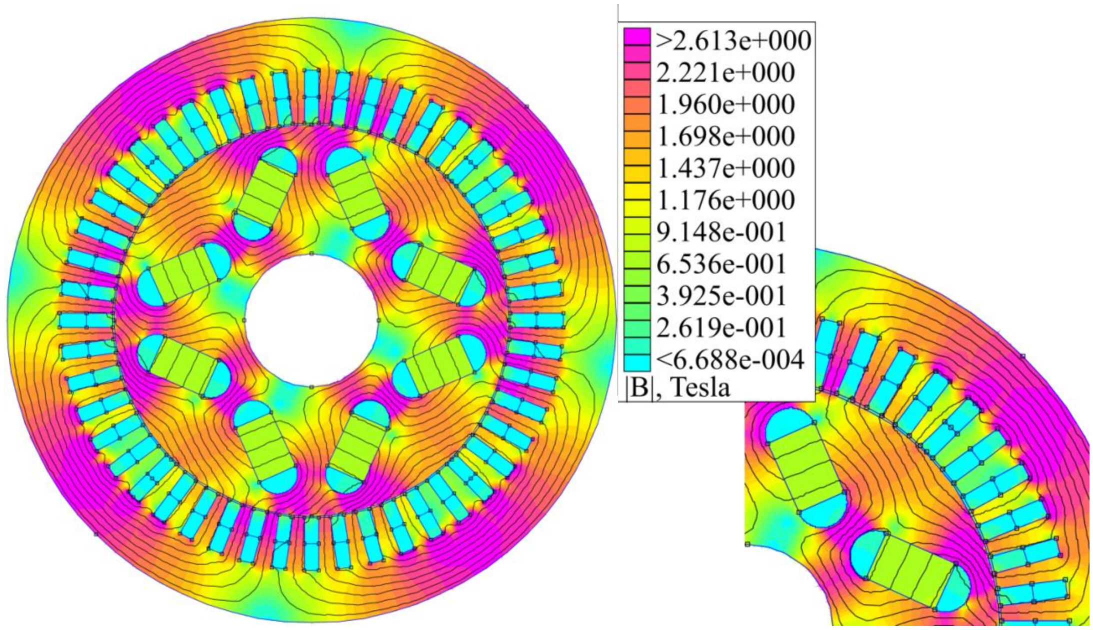
The currents in the stator windings should be changed rationally in the intervals from $-1{,}2\, I_{\phi \max}$ to $1{,}2\, I_{\phi \max}$ with a step of $(0.2..0.35)\, I_{\phi \max}$, where $I_{\phi \max}$ – maximum value of the current in the phase.

According to the results, the values of the flux linkage of the stator phases and the electromagnetic torque of the motor were obtained using the methods [8 - 10] and the FEMM software package [8, 10].

The results of the magnetic field calculation are shown in Fig. 4 – for a non-sectioned rotor, and Fig. 5 – for a non-sectioned one.

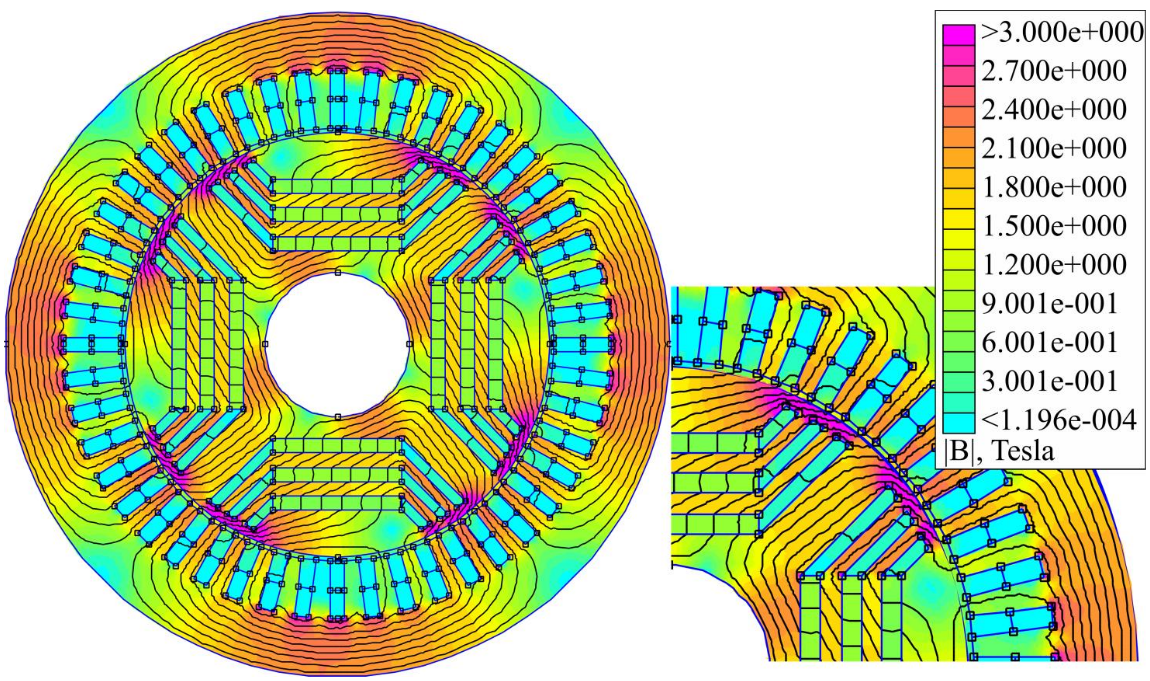
Regression analysis of the obtained flux linkage dependencies will be carried out using a special polynomial function. When choosing the second type, the following tasks were mastered:

- the proposed function should have a fairly simple form of analytical partial derivatives in all coordinates;



a b

Fig. 4 Results of calculation of the magnetic field of a synchronous jet motor with non-sectioned permanent magnets in nominal mode: a – general view; b – zone of permanent magnets.



a b

Fig. 5 Results of calculation of the magnetic field of a synchronous jet motor with sectioned permanent magnets in nominal mode: a – general view; b – zone of permanent magnets

- the function and its derivatives in the angle of rotation of the rotor at the beginning and end of the interval on which the approximation is performed should be the same;

- the dependence of the proposed function should take into account changes in currents not only of its own, but also of neighboring phases.

Taking into account the above, the proposed regressive function has the form

$$\Psi_k = w\sum_{l=1}^{m}\left[\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n} aa_{ijk}\cdot i_1^i\cdot i_2^j\cdot\ldots\cdot i_M^k\right)\times\right.$$
$$\left.\times cos(\,pl\gamma\,)+\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n} ab_{ijk}\cdot i_1^i\cdot i_2^j\cdot\ldots\cdot i_M^k\right)sin(\,pl\gamma\,)\right]+$$
$$+\,w\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n} ac_{ijk}\cdot i_1^i\cdot i_2^j\cdot\ldots\cdot i_M^k \quad , \qquad (14)$$

where w– number of turns of a phase, p – number of pole pairs for the motor, $aa_{ijk}, ab_{ijk}, ac_{ijk}$, $ba_{ijk}, bb_{ijk}, bc_{ijk}$, $ca_{ijk}, cb_{ijk}, cc_{ijk}$ – coefficients of the polynomial for phases 1, 2 and 3, respectively, determined using Chebyshev polynomials on a set of equidistant points; the number of turns of the winding of one phase; the angular frequency of the first harmonic of the flux.

The function of the motor torque has the form

$$M_m = \sum_{l=1}^{m}\left[\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n} ma_{ijk}\cdot i_1^i\cdot i_2^j\cdot\ldots\cdot i_M^k\right)\times\right.$$
$$\left.\times cos(\,pl\gamma\,)+\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n} mb_{ijk}\cdot i_1^i\cdot i_2^j\cdot\ldots\cdot i_M^k\right)sin(\,pl\gamma\,)\right]+$$
$$+\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n} mc_{ijk}\cdot i_1^i\cdot i_2^j\cdot\ldots\cdot i_M^k \quad , \qquad (15)$$

where $ma_{ijk}, mb_{ijk}, mc_{ijk}$ – coefficients of the polynomial determined by the Chebyshev method on a set of equidistant points.

To determine the coefficients of the approximating polynomial, it is proposed to use a method based on Chebyshev polynomials on a set of equidistant points [4, 5].

The rational order of the regression model describing the digital experiment is proposed to be determined based on the evaluation of the approximation results and its subsequent comparison with the experimental results using the maximum deviation criterion.

The results obtained from the regression analyses indicate that with an increase in the content of higher harmonics in the polynomial, the error value decreases. The approximation with n=3 and l=6 for the two considered designs is rational, while the maximum deviation is up to 2.62 % for a non-sectioned rotor and 2.79% for a sectioned one, which is acceptable.

Thus, based on the results of magnetic field calculations and subsequent regression analysis, polynomial dependences of the derivatives of flux linkages on current and angular coordinate were obtained, which make it possible to identify a generalized mathematical model of a synchronous traction motor.

Conclusions.

1. Based on the generalized mathematical model for electromechanical systems, a mathematical model of a synchronous traction motor of a metro car has been developed. The model is based on solving the Lagrange equation for an electromechanical system, taking into account the geometric features of the magnetic system of the rotor and stator of the motor, as well as the nonlinearity of the magnetic system.

2. A special regressive approximating function has been developed, the feature of which is the following: a simple form of analytical partial derivatives in all coordinates, the function and its derivatives in the angle of rotation of the rotor at the beginning and end of the interval on which the approximation is made are equal; the dependence of the proposed function must take into account changes in currents not only of its own, but also of neighboring phases.

3. According to the results of numerical experiments on the calculations of the magnetic field by the finite element method in the plane-parallel formulation of the problem and subsequent regression analysis, five-dimensional polynomial dependences of the derivatives of flux linkages and torque on current and angular coordinate were

obtained, which make it possible to identify a generalized mathematical model of a synchronous jet traction motor, both with a non-sectioned and sectioned rotor, for a metro car.

4. When conducting regression analysis, the rational order of the function approximating the flux linkage of the motor phases and its derivatives, as well as the electromagnetic torque, was determined for the first time. For a synchronous jet motor with permanent magnets, the following were determined: the number of harmonics is 7, the degree of the polynomial is 3 with a maximum deviation of 2.62% for a non-sectioned rotor and 2.79 for a sectioned one.

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Надійшла до редакції 18.01.2025 р.