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DETERMINATION OF THE ENERGY-OPTIMAL TRAJECTORY OF A METRO CAR BASED ON SOLVING A CONDITIONAL MINIMIZATION PROBLEM

Abstract. *The paper proposes a method for determining the optimal trajectory of movement along a section of the metro car track to apply a method based on solving the problem of determining the optimal values of the speed limit for each section of the path. It is proposed that the energy consumed from the intermediate circuit can be chosen as a criterion for solving the conditional minimization problem. However, for the full identification of the parameters of the problem, it is necessary to determine the efficiency dependencies in the electric motor and traction drive as a function of the torque on the shaft and the rotational speed. An algorithm is proposed for determining the efficiency dependencies of the traction drive and traction synchronous motor, which takes into account power losses from higher harmonics of the traction inverter.*
Keywords: *subway car; traction drive; optimal trajectory of subway train movement; traction motor efficiency, conditional minimization*

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ВИЗНАЧЕННЯ ЕНЕРГООПТИМАЛЬНОЇ ТРАЄКТОРІЇ РУХУ МЕТРОВАГОНУ НА ПІДСТАВІ ВИРІШЕННЯ ЗАДАЧІ УМОВНОЇ МІНІМІЗАЦІЇ

Анотація. *У роботі запропоновано якості методу визначення оптимальної траєкторії руху по ділянці колії метровагону застосувати метод на підставі вирішення задачі визначення оптимальних значень обмеження швидкості за кожною ділянкою шляху. Запропоновано енергію,* *що споживається з проміжного контуру можливо обрати у якості критерію вирішення задачі умовної мінімізації. Однак для повної ідентифікації параметрів задачі необхідно визначити залежності ККД у електродвигуні та тяговому приводі як функції моменту на валу та частоти обертання. Запропоновано алгоритм визначення залежностей ККД тягового приводу та тягового синхронного двигуна, що враховує втрати потужності від вищих гармонійних тягового інвертора.*

Ключові слова: *вагон метрополітену; тяговий привод; оптимальна траєкторія руху метропоїзду; ККД тягового двигуна, умовна мінімізація*

Introduction.

To solve the problem of optimal motion, it is proposed to use optimization based on the criterion of minimizing energy consumption. This approach allows us to determine the optimal law of change of maximum speeds on different sections of the track along which the train moves. Further, based on the solution of the problem of multiparameter conditional minimization, a schedule of train movement on the track section is constructed, taking into account the restrictions on the time of movement and the operating modes of the traction drive, in particular, on the power and conditions of wheel-rail adhesion. The results of the calculations make it possible to determine the requirements for the use of energy storage systems in subways

Analysis of previous research.

The theory of optimal control remains one of the most dynamically developing and relevant areas of modern science. It serves as a powerful mathematical tool for solving complex control problems, including the regulation of the movement of electric rolling stock in railway transport systems. As emphasized in [1], the theory of optimal control is a mathematical discipline with wide application in science and technology. The basis of control theory is the optimization problem [2], where the control variable is considered as an abstract entity, the goal of which is to maximize expected benefits (or minimize costs) over a given time horizon. A clear example of this principle can be observed in train traffic control systems. Considering the problem of controlling electric rolling stock on a certain section of track, the goal is to ensure that the train moves according to a specified movement program, arriving at the final station as close as possible to the scheduled time. The movement program consists of a sequence of actions, each of which is associated with certain costs - primarily path costs (energy consumed by the electric traction system) and terminal costs (deviation from the schedule). Optimal control in this sense involves choosing a sequence of control commands that direct the electric rolling stock to its destination with minimal energy consumption. If (x) denotes the state space of the system (e.g., position and velocity), the optimal control law can be expressed as a function that depends on both the current state of the system and time [2, 3].

A significant problem in stochastic control problems is their computational complexity, which increases exponentially with the increase in the number of possible states of the system. This is a result of the need to represent the state of the system in a discrete coordinate space, which leads to a large number of calculation points and an increase in computational resources [2]. To solve this problem, the Hamilton-Jacobi-Bellman equation is used, in particular, through the inverse Bellman approach, which allows obtaining an optimal solution with a feasible amount of calculations. This approach is especially effective for discrete transformations of systems in real time [2]. Given that the optimal control problem is formulated in continuous time, its implementation and interpretation in real time are of critical importance.

Dynamic programming serves as a fundamental tool for optimizing multi-stage processes, such as the movement of a train along a track segment. At each stage (or section), a partial optimization problem is solved using functional analysis methods or Lagrange multipliers. The strategy for obtaining the optimal solution involves taking into account the state of the system at the previous stage and selecting the most effective control parameters for the next. Among the dynamic programming approaches, the forward and backward pass methods, as well as methods derived from the Bellman optimality principle, are most often used in transport problems [5,6]. According to [7], although continuous-time problems can be solved using classical approaches such as the Lagrangian method and nonlinear programming, discretization of the problem — by dividing time or distance into a finite number of intervals — allows the development of real-time or spatial models suitable for use as expert systems in electric rolling stock management. In this context, the time domain is segmented into a limited number of discrete intervals.

The research presented in [8,9,10] shows that the straight-through method is the most computationally efficient. Its algorithm for determining the optimal train operation mode involves sequentially calculating the speed values at the endpoints of each section. These values determine the slope of the tachogram and the initial conditions for finding the slope on the next section. However, dividing the route into only three sections with constant traction does not properly take into account changes in the track profile or the nonlinear relationship between traction and train speed. Increasing the level of discretization, as proposed in [7], significantly increases the complexity of the mathematical model and the number of computational iterations required to solve the traffic optimization problem. This effect is especially pronounced on sections with complex track profiles or speed restrictions. Therefore, the determination of the operation modes in this study is carried out using multi-parameter constrained minimization methods.

Problem statement.

The aim of the work is to determine the optimal trajectory of the metro car on the sections of the track between stations, taking into account the track profile and the radii of curved sections.

Research results.

Objective function for determining the optimal parameters of the optimal movement of a metro car along a section of the track. One way to solve the problem of determining the optimal trajectory of movement along a section of the track is to solve the problem of determining the optimal values of the speed limit for each section of the track, which is given in [10]. Let us consider the main methods that form the basis of this approach.

Unconditional optimization methods are designed to find the extremum of an objective function with several variables f(x), where $x = (x_1, x_2, \ldots, x_n)^T$is a column vector of real variables, which can also be considered as a point in n-dimensional space $x \in R^n$. Here and further n is the number of variables, which determines the dimension of the parameter vector.

Vector $x = v_{max}$ consists of components of maximum speeds along the route section. Route sections should be rationally selected within the route with the same parameters of traffic resistance and requirements for the established maximum speeds or modes of metro train traffic.

Thus, the current speed of the metro car on the section of the track must comply with the conditions

$$V(t) \leq \mathrm{V}_{max}\,(S(t)), \tag{1}$$

where S is the distance traveled, $t$is the time of movement.

As an efficiency criterion, the value of energy consumption during the movement of the train along the track section is proposed, which can be determined based on the results of solving the traction problem.

Let us consider the main steps of solving the traction problem. When creating a model of the movement of a metro train on a section of the track, it is assumed that the train is modeled as a chain of solid bodies connected by an absolutely rigid connection. This increases the accuracy of calculations of the forces of resistance to movement [12]. The system of equations of motion has the form

$$\begin{cases} \frac{\mathrm{dV}}{\mathrm{dt}} = \frac{\xi}{\rho}(\mathrm{f_L} - (\mathrm{w_L} + \mathrm{w_W}) - \mathrm{b}) \\ \frac{\mathrm{dS}}{\mathrm{dt}} = \mathrm{V} \end{cases}, \tag{2}$$

ξ– coefficient that takes into account units of measurement, ρ– coefficient that takes into account rotation of the crew unit components, f_L– specific tangential force of the locomotive in traction or electrodynamic braking mode; w_L– specific resistance force of motor cars; w_W– specific resistance force of non-motor cars; *b*– specific braking force of pneumatic brakes.

The specific tangential force of the locomotive in traction or electrodynamic braking mode was determined by the expression

$$f_L = \frac{F_L}{\sum_{k=1}^{s} M_{Lk} + \sum_{j=1}^{n} M_{Wj}}, \tag{3}$$

where F_L is the tangential force in traction or electrodynamic braking mode; M_{Lk} is the mass of the locomotive section; s is the number of locomotive sections; M_{Wj} is the mass of the wagon, n is the number of wagons.

The tangential traction force of a metro car can take any value in the traction domain [3] .

The specific resistance to the movement of the locomotive and wagons was determined by the expression

$$w = w_o + w_i + w_r, \tag{4}$$

where w_o– the main specific resistance to movement, w_i– the additional specific resistance to movement from the slope, w_r– the additional specific resistance to movement from movement along the curve, w_p– the additional specific resistance to movement from movement by cars.

Calculation formulas for determining resistivity, as well as recommendations for their application, are presented in [3].

As mentioned earlier, to improve the accuracy of the drag force calculations, the train was modeled as a series of rigidly connected bodies. According to this approach, the drag caused by gradients and curve motion was calculated separately for each car or section of the metro train. A car was considered to be completely located on a given section of track when its center of mass was within that section.

The metro train movement model is supplemented with a traction force regulator in the form of

$$F = \begin{cases} F_L, \quad V < (V_{max} - \Delta V) \\ 0, \quad (V_{max} - \Delta V) \le V < V_{max}, \\ -F_L, \quad V > (V_{max} + \Delta V) \end{cases} \tag{5}$$

where $V_{\max}$is the permissible speed of movement, $\Delta V$is the zone of “insensitivity” [10]
.

In this case, the intensity of the change in the thrust force depends on the current speed and was determined by the expression [11]

$$\frac{\Delta F}{\Delta t} = \begin{cases} \left(\frac{\Delta F}{\Delta t}\right)_{\max}, & V < V_{\min} \\ k_F \left(\frac{\Delta F}{\Delta t}\right)_{\max}, & V_{\min} \leq V < (V_{\max} - \Delta V) \end{cases}, \tag{6}$$

where $V_{\min}$is the minimum regulation speed, $\left(\frac{\Delta F}{\Delta t}\right)_{\max}$is the maximum intensity of change in the tangential force speed, $k_F$is the intensity reduction coefficient.

In the electrodynamic braking mode, the increase in tangential force occurs with maximum intensity. This description of the change in tangential force in a certain sense corresponds to manual control of an electric locomotive.

The tangential power was determined by the expression

$$P_L = F_L V \tag{7}$$

Thus, expressions (1)-(7) constitute a mathematical model of train movement on a section of the track.

The power consumed from the intermediate circuit by one traction inverter is determined by the expression

$$P_{DC1} = r\left(\frac{P_T}{\eta_{tp}} + \Delta P_{GB}\right) \tag{8}$$

where r– the number of traction electric motors powered by one inverter ; P_T– tangential power, ΔP_{GB}– losses in the traction gearbox, ; η_{tp}– efficiency of the traction drive (further in the work this efficiency takes into account the traction inverter and the traction motor).

Due to the fact that the synchronous traction drive considered in the work uses a power supply circuit with an individual inverter for each traction motor, the efficiency of the traction motor - inverter link is considered as the concept of "traction drive efficiency".

Taking into account the provisions of [10], the losses in the traction reducer in the simplest form can be determined by the expression

$$\varDelta P_{GB} \approx \varDelta P_{GBnom}\frac{P_T}{P_{TMnom}} \tag{9}$$

where $\varDelta P_{GBnom}$– losses in the traction gearbox at the nominal power of the traction electric motor;

P_{TMnom}– nominal power of the traction electric motor.

Losses in a traction motor depend significantly on its operating mode. To simplify calculations, it is advisable to calculate the efficiency of the electric motor and traction drive in advance as a function of the torque on the shaft. and rotation frequency $\eta = f(M \quad, n \quad), \eta_{tp} = f(M \quad, n \quad)$.

The torque of the electric motor is calculated by the expression

$$M \quad = \frac{F_{KP}D_{KP}}{2\mu_{GB}} + \frac{30}{\pi}\frac{\varDelta P_{GB}}{n_{TM}} \tag{10}$$

where $F_{KP}$is the tangential force realized by one axle, $D_{KP}$is the diameter of the metro car wheel , $\mu_{GB}$is the gear ratio of the traction reducer, $n_{TM}$is the speed of the electric motor, which is determined by the expression

$$n \quad = \frac{1000\mu_{GB}V}{60\pi D_{KP}} \tag{11}$$

The calculation of losses in the traction motor was performed according to [10]. It should be noted that when the traction motor is powered by a voltage inverter, losses from higher harmonics of current and voltage occur in it. To calculate them, it is necessary to know the spectral composition of the inverter voltage and the dependence of the motor resistance on frequency, which is determined according to section 3.

The total power consumed for traction is determined by the expression

$$\mathrm{P_{DC}} = \mathrm{P_{DC1}}\frac{\mathrm{N_D}}{\mathrm{r}} \tag{12}$$

The energy consumed from the intermediate circuit was determined by the expression

$$\mathrm{E_{DC}} = \int_0^{\mathrm{T}} \mathrm{p_{DC}(t)dt} \tag{13}$$

where $p_{DC}(t)$is the dependence of the power consumed from the intermediate power circuit, $T$is the duration of the movement.

Thus, the energy consumed from the intermediate circuit can be chosen as a criterion for solving the conditional minimization problem. However, for a complete identification of the parameters of the problem, it is necessary to determine the dependences of the efficiency in the electric motor and traction drive as a function of the torque on the shaft and the speed $\eta = f(M\quad, n\quad)$, $\eta_{tp} = f(M\quad, n\quad)$.

To determine the dependences of the efficiency of the traction drive and the engine, a method based on [11,12] was chosen.

The results of calculating the dependencies are shown in Fig. 1.

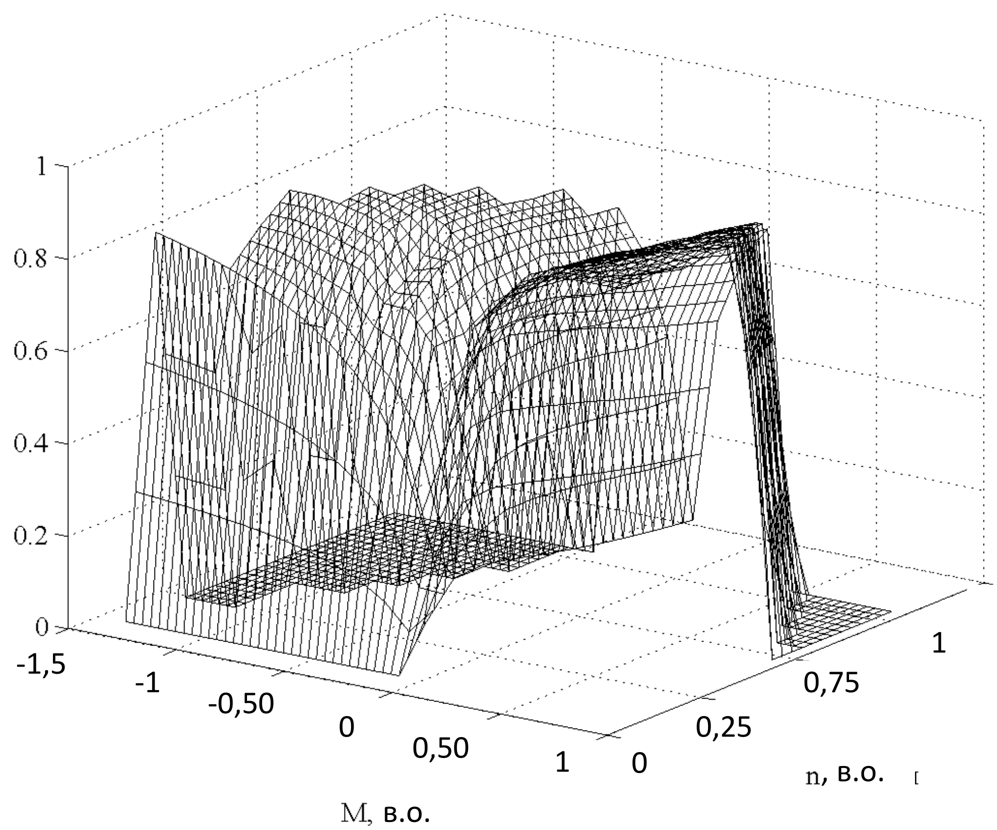

Fig. 1– Optimal value of drive efficiency

Thus, based on the results of the calculations, the dependences of the drive efficiency on the torque on the shaft and the speed of the traction motor were determined $\eta = f(M\quad, n\quad)$.$\eta_{tp} = f(M\quad, n\quad)$

The dependencies of the efficiency of the motor and drive have similar forms, however, the efficiency of the motor, which is part of the efficiency of the drive, is slightly higher, which is due to the inclusion of losses in the semiconductor converter in the efficiency of the drive.

The dependencies decrease to zero in the near-zero torque region, which is due to the reduction in drive torque and power. Also, the efficiency has limitations in the maximum power region at high values of the rotational speed and torque modulus on the motor shaft.

Setting constraints when solving the problem of conditional minimization of train movement along a section of the track. To formulate a minimization problem when calculating traffic, it is necessary to introduce restrictions that determine the traffic regime on the site.

The time of train movement along the track section is set as a constraint when solving the problem

$$t_{\min} < t_{end} < t_{\max}, \qquad (14)$$

where $[t_{\min}, t_{\max}]$is the interval that specifies the permissible train movement time on the track section.

The travel time is determined by solving the problem of the movement of a metro train on a certain section of the track, represented by a system of two differential equations (2) and (6). This system is solved by the fourth-order Runge-Kutta method under zero initial conditions.

The permissible range of train travel time can be reasonably established based on the given average speed on the section, which is determined by the traffic schedule and technological processes of the transport enterprise:

$$\begin{aligned} t_{\min} &= 0{,}99\,\frac{(X_{\Sigma}-X_{tr})}{v_{sr}}, \\ t_{\max} &= 1{,}01\,\frac{(X_{\Sigma}-X_{tr})}{v_{sr}}, \end{aligned} \qquad (15)$$

where X_{Σ} is the total length of the line section, v_{sr} is the average speed of movement along the section, which rotates according to the passenger traffic load schedule in the subway.

In addition, the parameters are subject to restrictions set by the train schedule on the section of the track. This procedure must be followed on all sections.

$$v_{\min} < \dot{x}_u < u_i < v_{\max}, \tag{16}$$

where $v_{\min}$, $v_{\max}$– the minimum and maximum speed on the track section.

From the safety conditions for movement in the metro, the maximum speed of movement on directional sections of the track was chosen $v_{\max}$= 80 km/ h , and the minimum is $v_{\max}$= 1 km/h.

Thus, a list of constraints imposed when solving the conditional minimization problem for a given average speed of the metro car has been formed .

Solving the problem of conditional minimization of train movement along a section of the track

For each separate section of the inter-station run, it is proposed to carry out a separate solution to the problem. Thus, the solution to the problem consists of solving individual traffic problems on the inter-station runs.

To optimize train movement parameters, it is proposed to use the MATLAB software environment (USA) in combination with the Optlab optimization package (Ukraine) [7,8,9].

The conditional minimization problem is solved by the Weyl method with further refinement of the obtained results by the Nelder-Mead method . These approaches have demonstrated the highest efficiency in solving the problem, since they require the smallest number of estimates of the objective function.

Weyl method search is determined by the vector of given speed values along individual sections of the track. And the starting point for the Nelder - Mead method is the results of the Weyl method search .

The progress of solving the problem for the sections of the Saltivska line of the Kharkiv metro from the "Kyivska" station to the "Akademika Barabashova " station (Fig. 2 and Fig. 3) for a given average speed of 20 km/ h at full load of the metro train according to the Weil method is shown in Fig. 2, and according to the Nelder - Mead method –Fig. 3, respectively.

Having analyzed the curves of train movement along optimal trajectories on the inter-station sections of the Saltivska line of the Kharkiv metro, the following was determined (Fig. 4). Due to the small distance between stations, which does not exceed 2400 m, the movement of the metro train is optimally carried out in modes close to the traction forces and braking close to the maximum values of the traction motor efficiency. Partial traction force is observed only in minor sections while maintaining the specified maximum speed. On a number of sections, the set speed does not reach the limits for v_{max}, which is due to the limitation of wheel-rail adhesion. But the value v_{max}determines the maintenance of the change in the intensity of the traction force (6) at the level necessary for train movement.

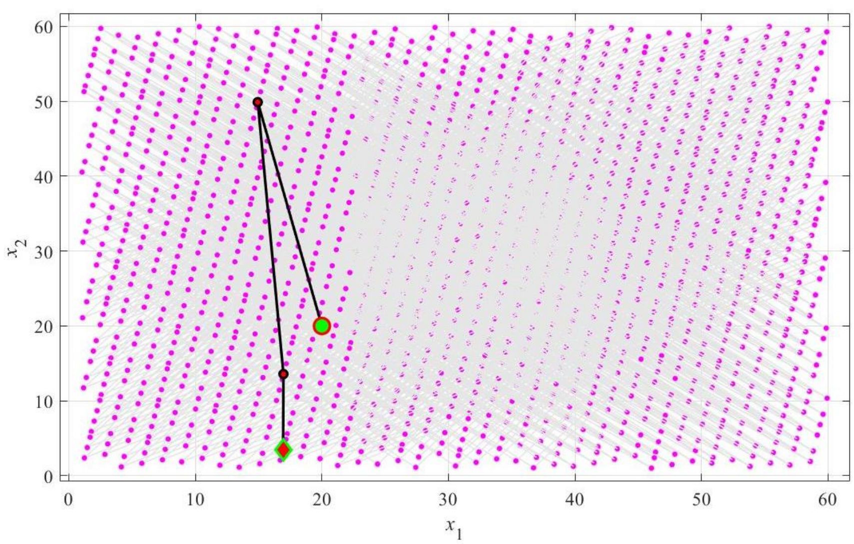

Fig. 2 – Trajectory of search by the Weyl method for optimal values of given speeds along sections 1,2 of the path from the station "Kyivska" to the station "Akademika Barabashova ": black line – optimal trajectory, gray line – steps that are not optimal, circle – starting point of search, rhombus – end point of search.

Conclusions. 1. As a method for determining the optimal trajectory of movement along the section of the metro car track, apply a method based on solving the problem of determining the optimal values of the speed limit for each section of the path.

It is proposed that the energy consumed from the intermediate circuit can be chosen as a criterion for solving the conditional minimization problem. However, for the full identification of the parameters of the problem, it is necessary to determine the dependences of the efficiency in the electric motor and traction drive as a function of the torque on the shaft and the speed $\eta = f(M\ \ ,n\ \ ).\eta_{tp} = f(M\ \ ,n\ \ )$.

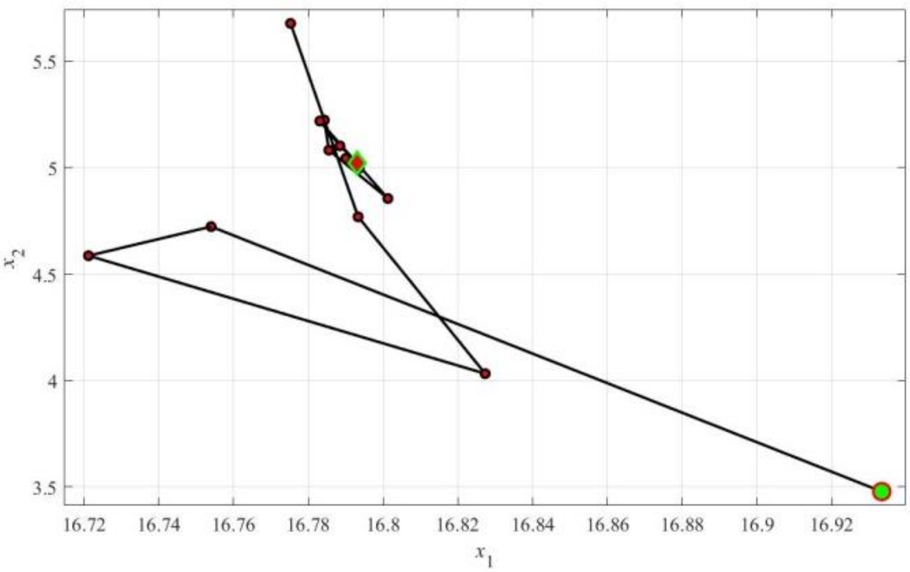

Fig. 3 – Search trajectory using the Nelder - Mead method of optimal values of set speeds along sections 1,2 of the path from the station "Kyivska" to the station "Akademika Barabashova ": black line – optimal trajectory, circle – starting point of search, rhombus – end point of search.

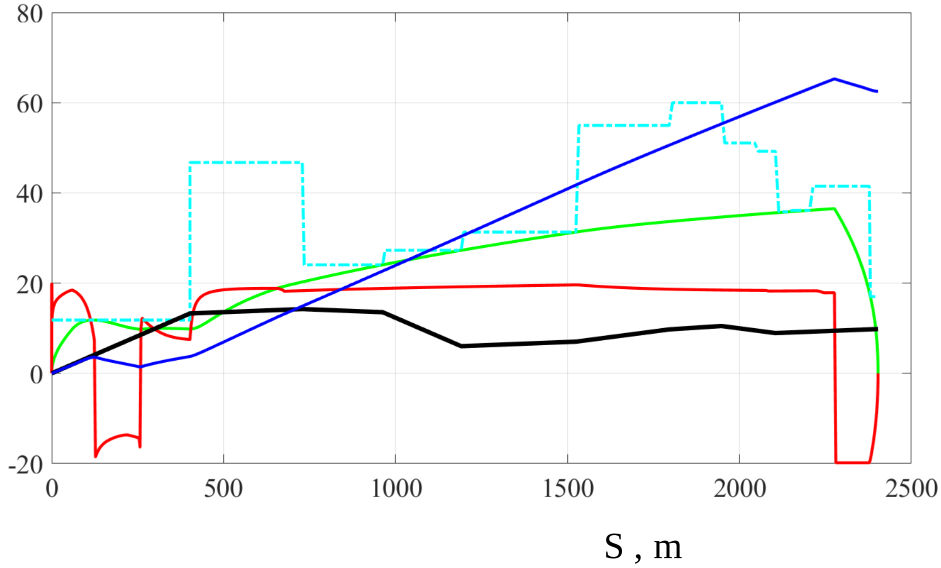

Fig. 4 – Results of solving traction problems according to the optimal schedule of movement of track segments from “Kyivska” to the station “Akademika Barabashova ” : turquoise line – set train speed on the track section, km/ h , green line – train speed, km/ h , red line – tangential traction force, N · 10 4 , blue line – energy consumption consumed from the intermediate circuit kW · h · 0.5, black line – directed track profile, m

An algorithm for determining the dependences of the efficiency of the traction drive and the traction synchronous motor is proposed, which takes into account power losses from higher harmonics of the traction inverter. The dependences of the efficiency of the traction motor and the drive have similar forms, however, the efficiency of the motor, which is part of the efficiency of the drive, is slightly higher, which is due to the inclusion of losses in the semiconductor converter in the efficiency of the drive.

The dependencies are reduced to zero in the torque zone close to zero, which is due to a decrease in the torque and power of the drive. Also, the efficiency has limitations in the maximum power zone at high values of the rotational speed and torque modulus on the engine shaft.

A list of constraints imposed when solving the conditional minimization problem for a given average speed of the metro car has been formed.

The problem was solved using the Weyl method with further refinement of the results obtained by the Nelder-Mead method. These approaches demonstrated the highest efficiency in solving the problem, since they require the smallest number of estimates of the objective function.

2. Having analyzed the curves of train movement along optimal trajectories on the interstation sections of the Saltivska line of the Kharkiv metro, the following was determined. Due to the small distance between stations, which does not exceed 2400 m, the movement of the metro train is optimally carried out in modes close to the traction forces and braking close to the maximum values of the efficiency of the traction engine. Partial traction force is observed only in small sections while maintaining the specified maximum speed. When stopping the metro train, it is optimal to use regenerative braking, which is characterized by a negative value of the traction force and a decrease in energy consumption for the train's traction. The reduction in energy consumption determines the level of energy that can be used for accumulation in the on-board energy storage. It is the maximum level of reduction that determines the energy intensity of the on-board energy storage. The results of determining the recuperation energy for inter-station runs are determined.

3. It has been established that the power of the traction drive, which determines both the dynamics of train movement and the requirements for the storage device, is at a level close to the maximum values for the traction drive – 2199 kW, which is due to the high efficiency of the drive in modes close to the maximum maximum power mode.

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