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PROCESS ANALYSIS IN SINGLE-PHASE HALF-BRIDGE INVERTER BASED ON EXTENSION OF DIFFERENTIAL EQUATIONS

In the artical a method for calculation of steady-state processes in inverters with sinusoidal outputs is considered. The method is based on expanding of a differential equation with one time variable to a partial differential equation with two independent time variables and on using the two dimensional Laplace transform. The obtained solution is determined a periodical steady-state process in a domain of two variables of time.

Key words: steady-state process, inverter, expanding differential equation, two dimensional Laplace transform.

У статті представлено метод розрахунку вимушених процесів в інверторі з вихідною синусоїдальною напругою. Метод базується на розширенні диференційного рівняння з однією змінною часу у рівняння з частинними похідними з двома незалежними змінними часу та застосуванні двомірного перетворення Лапласа. Отримане рішення визначає вимушений періодичний процес у просторі двох змінних часу.

***Ключові слова** – вимушений процес, інвертор, розширення диференційного рівняння, двомірне перетворення Лапласа.*

Introduction

Single-phase half-bridge inverters with sinusoidal outputs formed by a pulse-width modulation (PWM) are widely used in different energy systems. One of methods used to form harmonic output signals is based on comparing of a triangular carrier voltage with a low-frequency sinusoid. Processes in a load of the inverter which is shown in Fig. 1 are described by complicated functions. In order to analyze such processes a method based on using of the double Fourier series [2] is used. This method describes an output voltage of the inverter in form of a double Fourier series. A further analysis is provided in the domain of one variable of time.

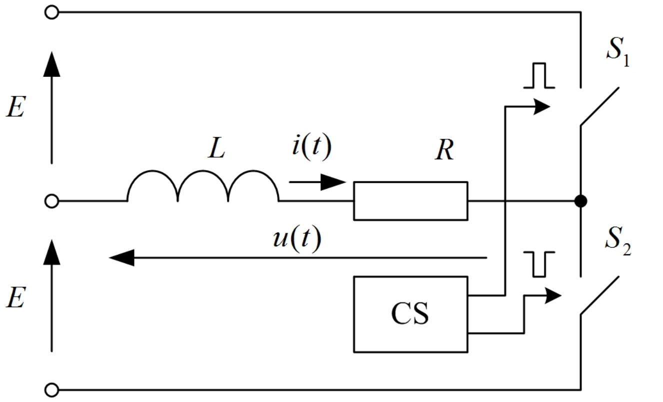

Fig.1. The topology of the inverter

In order to find a periodical steady-state process in a circuit of the inverter one can use the method of expanding a differential equation in a domain of two independent variables of time [1].

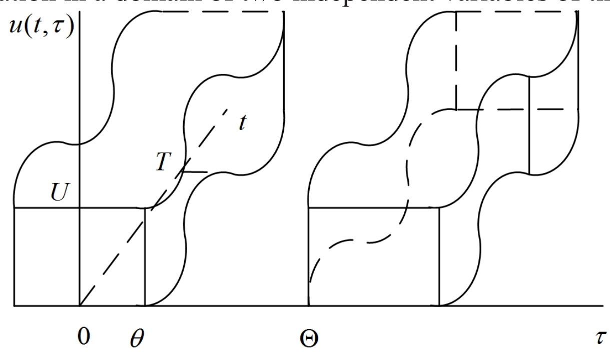

Fig. 2. Waveform of the output voltage of inverter in the domain of two time variables

In that case, as it has been shown in [4], a periodical steady-state process can be calculated in the circuit of the inverter with PWM formed by comparison of a sawtooth ramp voltage and a sinusoidal voltage.

The subject of this article is to spread the method for another case, when sinusoidal output voltage is obtained by a double side modulation of the output voltage as shown in Fig.2. The method is based on an expanding of the differential equation by introducing an additional independent time variable and the use of the two dimensional Laplace transform [3]. The steady state process calculated by the use of obtained expressions is compared with calculations of the differential equation with partial derivatives.

Mathematical model

We find a steady-state current in the inverter shown in Fig. 1. A control circuit CS generates control signals by comparing the sawtooth ramp voltage and sinusoidal voltage. These signals turn on and off switches S_1 and S_2 in the opposite phase. We assume that switches are ideal and the load is linear. Processes in the load of the inverter are described by the differential equation

$$L\frac{di(t)}{dt} = -Ri(t) + u(t)\,, \qquad (1)$$

where: $i(t)$ is a current, $u(t)$ is an output voltage of the inverter.

The output voltage is periodical in the domain of two time variables: $0 \le t \le T$, $0 \le \tau \le \Theta$, as shown in Fig.2. Therefore, it is expedient to expand the domain of the differential equation (1) from one independent time variable t to two independent time variables t and τ [3, 5] in following way

$$L\frac{\partial i(t,\tau)}{\partial t}+L\frac{\partial i(t,\tau)}{\partial \tau}=-Ri(t,\tau)+u(t,\tau). \qquad (2)$$

Transition from (1) to (2) is realized by the introduction of one more derivative with respect of the new time variable.

We use the two-dimensional Laplace transform [1]

$$F(s,q)=\int_0^\infty\int_0^\infty f(t,\tau)e^{-st-q\tau}d\tau dt$$

in order to convert the differential equation (2) into an algebraic equation.

Let $U(s,q)$ and $I(s,q)$ be images of the voltage $u(t,\tau)$ and current $i(t,\tau)$. Applying the two dimensional Laplace transform to (2) one obtains the following

$$L(s+q)I(s,q)=-RI(s,q)+U(s,q), \qquad (3)$$

We assume that boundary conditions equal zero, i.e. $i(t,0)=i(0,\tau)=0$. Solving (3) for $I(s,q)$ gives

$$I(s,q)=\frac{U(s,q)}{L(s+q)+R}. \qquad (4)$$

The voltage $u(t,\tau)$ is periodical

$$u(t,\tau)=u(t+T,\tau+\Theta),$$

so, by using the formula

$$F(s,q)=\frac{\int_0^T\int_0^\Theta f(t,\tau)e^{-st-q\tau}d\tau dt}{(1-e^{-sT})(1-e^{-q\Theta})}$$

one finds an image of a periodical function. Taking into account the form of the voltage $u(t,\tau)$, the image $U(s,q)$ is calculated as follows

$$U(s,q)=\frac{\widetilde{U}(s,q)}{(1-e^{-sT})(1-e^{-q\Theta})},$$

where

$$\widetilde{U}(s,q)=\int_0^T\int_0^\Theta u(t,\tau)e^{-st-q\tau}d\tau dt=$$

$$\int_0^T e^{-st}\int_{-\sigma-K\cos\omega t}^{\sigma+K\cos\omega t} Ue^{-q\tau}d\tau dt=\frac{U}{q}\int_0^T\left(e^{q(\sigma+K\cos\omega t)}-e^{-q(\sigma+K\cos\omega t)}\right)e^{-st}dt,$$

$\omega=\frac{2\pi}{T}$; U is a constant voltage that equals $2E$.

In order to find the original to (4) we calculate residues at singularity points of the image $I(s,q)$

$$s=-q-\frac{R}{L}, \qquad (5)$$

$$s_{\pm n}=\pm jn\omega,\ n=0,1,2,.., \qquad (6)$$

$$q_0^2=0,\ q_{\pm m}=\pm jm\Omega,\ m=0,1,2,.., \qquad (7)$$

where $\Omega=\frac{2\pi}{\Theta}$.

We do not take into consideration the integral

$$\int_0^T\left(e^{q(\sigma+K\cos\omega t)}-e^{-q(\sigma+K\cos\omega t)}\right)e^{-st}dt$$

since it has not singularity points. We also do not take into account the pole (5) since this pole defines a transient process.

Let us find the steady-state process for $I(s,q)$. Calculating the residue at the point $q^2=0$, $s=0$ we get

$$\lim_{s\to 0}\left\{ s\lim_{q\to 0}\left[\frac{d}{dq}\left(q^2 I(s,q)e^{q\tau}\right)\right]e^{st}\right\}=\frac{2\sigma U}{\Theta R}.$$

The obtained result corresponds to constant factor of double Fourier series. We shall not further take into consideration this value since a constant value of the inverter output voltage must be zero.

Now we calculate the residue at points $q^2=0$, $s=\pm j\omega$

$$i_{1,0}(t,\tau)=2\operatorname{Re}\lim_{s\to j\omega}\left\{(s-j\omega)\lim_{q\to 0}\left[\frac{d}{dq}\left(q^2 I(s,q)e^{q\tau}\right)\right]e^{st}\right\}=\frac{2KU\cos(\omega t-\varphi_1)}{\Theta\sqrt{(R^2+\omega^2L^2)}},$$

where $\varphi_1=arctg\frac{\omega L}{R}$.

This result corresponds to the first harmonic of the output current. The calculation of residues at points $q=0$, $s=\pm jn\omega$ for $n=2,3,..$, gives that they equal zero.

Calculating residues at points $s=0$, $q=\pm jm\Omega$ for $m=1,\,2,\,..$ one obtains as follows

$$i_{0,m}(t,\tau)=2\operatorname{Re}\lim_{s\to 0}\left\{s\lim_{q\to jm\Omega}\left[\left((q-jm\Omega)I(s,q)e^{q\tau}\right)\right]e^{st}\right\}=$$

$$\frac{2UJ_0(mK\Omega)}{m\pi Z_{0,m}}\cos\left(m\Omega\tau-\phi_{0,m}\right)\sin\left(m\Omega\sigma\right),$$

where $J_0(mK\Omega)$ is the Bessel function of the first kind; $Z_{0,m}=\sqrt{R^2+(m\Omega L)^2}$; $\phi_{0,m}=arctg\frac{m\Omega L}{R}$.

Calculating residues at points $s=\pm jn\omega$, $n=1,2,..$, $q=\pm jm\Omega$ for $m=1,\,2,\,..$ we get

$$i_{n,m}(t,\tau)=2\operatorname{Re}\lim_{s\to jn\omega}\left\{(s-jn\omega)2\operatorname{Re}\lim_{q\to jm\Omega}\left[\left((q-jm\Omega)I(s,q)e^{q\tau}\right)\right]e^{st}\right\}=$$

$$\frac{4UJ_n(mK\Omega)\sin(m\Omega\sigma+n\frac{\pi}{2})}{\pi m Z_{n,m}}\cdot\left\{\sqrt{R^2+(\omega nL)^2}\cos(n\omega t-\sigma_{n,m})\cos\left[m\Omega\tau\right]+\right.$$

$$\left.m\Omega L\sin(n\omega t+\varphi_{n,m})\sin\left[m\Omega\tau\right]\right\}$$

where

$$Z_{n,m}=\sqrt{R^4+L^4\left[(n\omega)^2-(m\Omega)^2\right]^2+2(LR)^2\left[(n\omega)^2+(m\Omega)^2\right]},$$

$$\varphi_{n,m}=arctg\frac{R^2+L^2(m^2\Omega^2-n^2\omega^2)}{2RLn\omega},\quad \sigma_{n,m}=arctg\frac{Ln\omega\left[R^2+L^2(n^2\omega^2-m^2\Omega^2)\right]}{R\left[R^2+L^2(n^2\omega^2+m^2\Omega^2)\right]}.$$

The steady-state current corresponding to solution of (2) has the form

$$i(t,\tau)=\sum_{\substack{n=0\\ m\neq 0}}^{\infty}\sum_{\substack{m=0\\ n\neq 0}}^{\infty}i_{n,m}(t,\tau).$$

In this expression the constant value of the current is not taken into account, since the output voltage of the inverter has not the constant component.

Results of calculation

Let us find the steady-state process for element values: $R=1\Omega$; $L=0{,}4\,mH$; $E=1V$; $T=20\,ms$; $\Theta=T/100\,ms$; $K=\ 0{,}133\Theta\,ms$; $\sigma=0{,}4\Theta\,ms$. The time waveform of the steady-state load current in the space of two time variables for $n=1,2,3,4$ and $m=1,2,3$ are shown in Fig. 3.

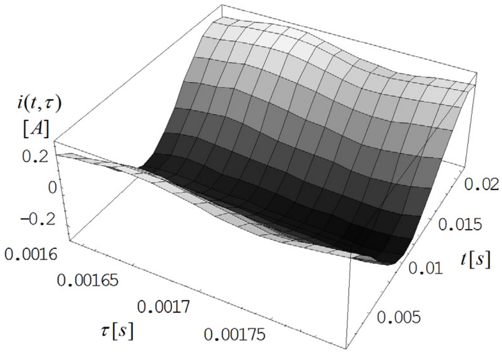

Fig.3. The steady-state current $i(t,\tau)$ in the domain of two time variables

In order to estimate the results obtained by the proposed method, one can calculate the transient process. After roughly three time constants, i.e. L/R, obtained results will correspond to the steady-state process. The time waveforms of the steady-state current for proposed method and after calculation the transient process are shown in Fig.4.

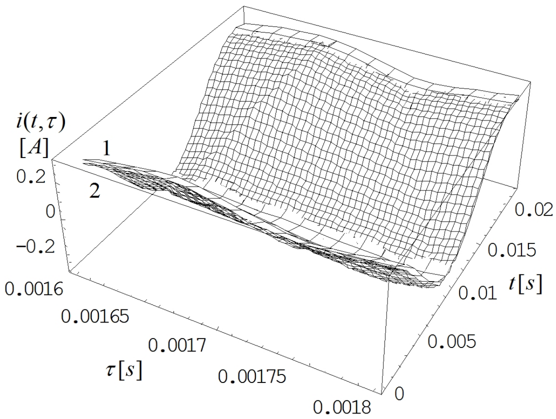

Fig.4. The steady-state currents $i(t,\tau)$ in the domain of two time variables

The surface 1 corresponds to results obtained by the proposed method and the surface 2 corresponds to results obtained after the calculation of the transient process. Comparing obtained results one can conclude that they well coincide.

The time waveforms of current $i(t)$ in the domain of one time variable ($t=\tau$) is shown in Fig.5.

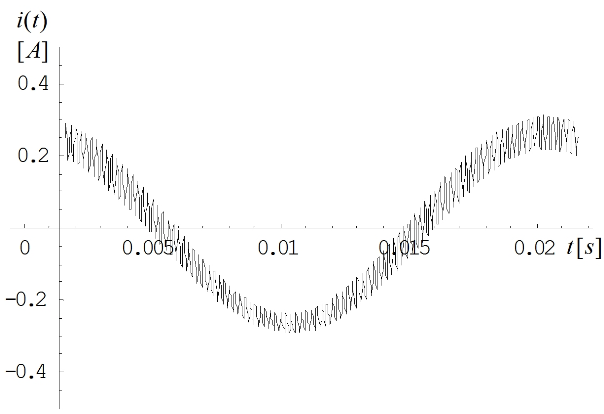

Fig.5. The steady-state current $i(t)$ for $t=\tau$

As one would expect the first harmonic dominates in the steady-state current.

Conclusions

In this paper the method for finding of steady-state processes in circuits of the inverter with the sinusoidal PWM control has been presented. For finding the periodical steady-state process the ordinary differential equation with one time variable has been extended to the partial differential equation with two time variables. The obtained equation has been solved by the use of the two-dimensional Laplace transform.

The calculated process obtained by the proposed method has been compared with the process obtained after the calculation of the transient process.

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В статье рассмотрен метод расчета установившихся процессов в инверторе с выходным синусоидальным напряжением. Метод основывается на расширении дифференциального уравнения с одной переменой времени до уравнения в частных производных с двумя независимыми переменными времени и применении двумерного преобразования Лапласа. Полученное решение определяет периодический установившийся процесс в пространстве двух переменных времени.

***Ключевые слова** – установившийся процесс, инвертор, расширение дифференциального уравнения, двумерное преобразование Лапласа.*